A POINT KERNEL MODEL FOR THE ENERGY DEPOSITED ON SAMPLES FROM GAMMA RADIATION IN A RESEARCH REACTOR CORE

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ABSTRACT

A basic safety requirement for a research reactor is the reliable estimation of the gamma heating of samples irradiated in the reactor core. A three-dimensional numerical code of gamma heating using a point kernel parameterization is developed. The heating due to $\gamma$-rays, produced from U-235-fission prompt and delayed, and from $(n, \gamma)$ reactions with the core materials is considered. The dose build-up due to the photons scattering on the core materials, as well as the energy absorption build-up in the sample, are also included, based on empirical relationships. The developed code, (GHRRC: Gamma Heating in Research Reactor Cores) is applied for the Greek Research Reactor (GRR-1) core. The required three-dimensional neutron fluxes are obtained with the neutronics code system XSDRNPM and CITATION. The macroscopic cross sections of U-235 fission and $(n, \gamma)$ reactions in the core materials are determined assuming a homogenized core. Comparisons of the computed gamma heating power distribution on a Fe sample, with measurements of the thermal neutron flux and the Fe sample temperatures in GRR1, show that GHRRC has a good qualitative behavior. GHRRC may easily be handled even by poorly experienced users.
I. INTRODUCTION

Heating from gamma radiation of irradiated sample materials is an issue of primary importance for the safety and the radiation protection of research reactors. Designing of the optimum conditions for a sample irradiation requires calculation of the energy that will be deposited on the target material. Several computational tools have been developed for gamma heating, based either on the point kernel approach such as in the MERCURE-5\textsuperscript{1} code, or on the Monte Carlo method such as in the TRIPOLI-4\textsuperscript{1} and MCNP\textsuperscript{2} codes. Approximate methodologies have also been reported, which estimate the deposited gamma energy either assuming mono-energetic photons and spatial zones where gamma ray production is uniform and isotropic\textsuperscript{3}, or based on experimental gamma dose rates and subsequent use of a correction factor, for converting dose-to-energy in various media\textsuperscript{4}.

A review of the related reports indicates that the accurate assessment of gamma heating of materials in nuclear reactor cores remains a matter of investigation. More recent computational works have been performed in the frame of the design of the future fusion reactor ITER (International Thermonuclear Experimental Reactor) and utilize mainly Monte Carlo codes combined with selected gamma kerma libraries\textsuperscript{5-7}. In this type of approach, the specification of the gamma kerma factors appears to be crucial\textsuperscript{8}. For example, in comparison with measurements of the ITER nuclear heating experiment, simulations with two alternative kerma libraries have produced overestimations of 20-30\% for the gamma energy deposited in a tungsten sample\textsuperscript{5}. In the same context, it is worth noting that in a frame of benchmark experiments for the tungsten neutron cross sections validation, analysis of the experimental results, using different kerma factors libraries, indicated gamma heating underestimations up to 40\% or overestimations up to a factor of four\textsuperscript{9}.

An improved gamma-heating computational method has also been suggested for fast reactors, basing the gamma flux calculation on a more accurate determination of the gamma source distribution, obtained from the computed neutron flux map; however despite the above modeling improvements, comparisons with measurements performed at the MASURCA facility of CEN Cadarache, indicated significant discrepancies between the predicted and measured values\textsuperscript{10}. 

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In the present work, a three-dimensional numerical code (GHRRC: Gamma Heating in Research Reactor Cores), based on a point-kernel parameterization, was developed to estimate the gamma heating of small samples inside a research reactor core, during operation. The model includes the prompt and delayed photons produced from the U-235 fission and the gammas produced by neutron capture -(n,γ) reactions- in the core materials. Empirical correlations are adopted for the dose build-up in the core and the energy absorption build-up in the irradiated sample\textsuperscript{11,12}. The required neutron fluxes are calculated using the neutronics code system XSDRNP\textsuperscript{13} and CITATION-LDI\textsuperscript{2}\textsuperscript{14} in a three-dimensional representation of the Greek Research Reactor (GRR-1) core. For the determination of the macroscopic cross sections for the U-235 fission and the (n,γ) reactions in the core materials, a homogenization of the core is performed. The (n,γ) reactions in the beryllium reflector blocks are omitted at this stage, since their contribution is expected rather small in the core. Preliminary computations verify that (n,γ) reactions in the beryllium reflector can be ignored when gamma heating is computed for a sample irradiated inside the reactor core.

The GHRRC code was applied to simulate the gamma heating of a Fe sample positioned at several depths of a central irradiation channel of the GRR-1 core. The present model is suggested as an easily handled computational tool, able to provide reasonable estimations for the gamma heating of materials irradiated in a reactor core during operation. The estimated error margin of the above prediction allows the Reactor Operator to pre-determine the irradiation conditions so that the sample temperature will safely remain well below the melting point during irradiation.

II. FORMULATION

Let a sample material placed at the position \( \vec{r} \) of the reactor core and a photon source of energy \( E \) at position \( \vec{r}_0 \) of the core. The sample’s dimensions are assumed small.

According to the linear attenuation model, the probability \( p(\vec{r},\vec{r}_0,E) \) that a photon of energy \( E \) found at the position \( \vec{r}_0 \) of the core crosses the distance \( |\vec{r} - \vec{r}_0| \) without being collided is \( \exp(-\mu(E)|\vec{r} - \vec{r}_0|) \), where \( \mu(E) \) is the attenuation coefficient for the monoenergetic γ-rays in core.
materials [cm⁻¹]. If the photons produced at the \( \vec{r}_0 \) position are assumed to move isotropically towards all directions in the core, their probability to reach the sample without being collided writes:

\[
p(\vec{r}, \vec{r}_0, E) = \frac{e^{-\mu(E)|\vec{r} - \vec{r}_0|}}{4\pi|\vec{r} - \vec{r}_0|^2}
\]  

(1)

\( \Pi(\vec{r}_0, E)d\vec{r}_0 \) being the production rate of monoenergetic photons of energy \( E \) per unit volume at the core position \( \vec{r}_0 \), the flux of these photons at position \( \vec{r} \) will be:

\[
F(\vec{r}, E) = p(\vec{r}, \vec{r}_0, E)\Pi(\vec{r}_0, E)d\vec{r}_0 = \frac{e^{-\mu(E)|\vec{r} - \vec{r}_0|}}{4\pi|\vec{r} - \vec{r}_0|^2} \Pi(\vec{r}_0, E)d\vec{r}_0
\]

(2)

If \( \mu_{ab}(E) \) is the absorption coefficient of the monoenergetic photons of energy \( E \) in the sample material, the rate of energy deposition per unit volume of the irradiated sample is given as:

\[
w'(\vec{r}, E)dE = \mu_{ab}(E) E F(\vec{r}, E)
\]

(3)

The rate of production of prompted and delayed photons, produced by fission at \( \vec{r}_0 \), is proportional to the local fission rate, that is, proportional to the local multigroup neutron flux \( \Phi(\vec{r}_0) \). Accordingly, the rate of fissions per unit volume at position \( \vec{r}_0 \) with neutrons of the energy group ‘\( n \)’ is given by \( \Sigma_{f,n}(\vec{r}_0)\Phi_n(\vec{r}_0) \). Further, \( X_n(E)dE \) being the probability that a photon of energy between \( E \) and \( E+dE \) results from fission-produced neutron at the energy group ‘\( n \)’, then the production rate of monoenergetic photons due to fission neutrons of the energy range ‘\( n \)’, is:

\[
\Pi(\vec{r}_0, E)d\vec{r}_0 = \Sigma_{f,n}(\vec{r}_0)\Phi_n(\vec{r}_0)X_n(E)dEd\vec{r}_0
\]

(4)

where
\[ \Sigma_{f,n}(\vec{r}_0) \text{ [cm}^{-1}\text{]} \], is the fission macroscopic cross section collapsed in the neutron energy group ‘\( n \)’ at the core position \( \vec{r}_0 \), and

\[ \Phi_n(\vec{r}_0) \text{ [cm}^{-2}\text{ s}^{-1}\text{]}, \] is the flux of neutrons in the energy group ‘\( n \)’ at core position \( \vec{r}_0 \).

To take also into account the gamma rays due to (n, \( \gamma \)) reactions in the core materials with neutrons of the energy group ‘\( n \)’, the product \( \Sigma_{f,n}(\vec{r}_0)X_n(E) \) must be increased by the sum over all nuclides ‘\( j \)’ included in the active reactor core \( S_n(\vec{r}_0,E) = \sum_j \Sigma_{j,n}(\vec{r}_0)Y_{j,n}(E) \), where:

\[ \Sigma_{j,n}(\vec{r}_0) \text{ [cm}^{-1}\text{]}, \] is the macroscopic cross section of (n,\( \gamma \)) reaction for nuclide ‘\( j \)’, with neutrons of the energy group ‘\( n \)’ at core position \( \vec{r}_0 \) and

\[ Y_{j,n}(E) \text{ [J}^{-1}\text{]}, \] is the spectrum of gamma rays due to (n,\( \gamma \)) reactions in nuclide ‘\( j \)’, with neutrons of the energy group ‘\( n \)’.

Using the notation \( A_n(\vec{r}_0,E) = S_n(\vec{r}_0,E) + \sum_{j,n}(\vec{r}_0)X_n(E) \), it arises from Eqs. (2 – 4) that the monoenergetic gamma rays due to fission induced by neutrons of the energy group ‘\( n \)’ at the position \( \vec{r}_0 \) of the reactor core, depose energy per unit volume of the irradiated sample with a rate:

\[
w'(\vec{r},E)dE = dE\mu_{eb}(E)\frac{e^{-\mu(E)|\vec{r}-\vec{r}_0|}}{4\pi|\vec{r}-\vec{r}_0|^2} A_n(\vec{r}_0,E)\Phi_n(\vec{r}_0)d\vec{r}_0
\]  

(5)

It is assumed that in the sample mass, all photons cover an average distance \( \bar{\ell} \) equal to the mean chord length\(^{15} \) of the sample, defined as \( \bar{\ell} = 4V_s/S_e \), where \( V_s \) and \( S_e \) are respectively the volume and the external surface of the sample; then, the integration of Eq. (5) over the sample’s volume gives:

\[
w(\vec{r},E)dE = \int_0^{\bar{\ell}} w'(\vec{r},E)dEdx = dE\frac{\mu_{eb}(E)}{\ell\mu_{a}(E)}\left(1-e^{-\mu_{a}(E)\bar{\ell}}\right)\frac{e^{-\mu(E)|\vec{r}-\vec{r}_0|}}{4\pi|\vec{r}-\vec{r}_0|^2} A_n(\vec{r}_0,E)\Phi_n(\vec{r}_0)d\vec{r}_0
\]  

(6)
where $\mu_a(E)$ in [cm$^{-1}$] is the attenuation coefficient of the monoenergetic photons of energy $E$ in the sample.

The dose build up in the core and the energy absorption build-up in the sample are respectively included by multiplying Eq. (6) with the build-up factors $B_{c}(\mu(E)|\vec{r} - \vec{r}_0|, E)$ and $B_{s}(\mu_a(E)|\vec{r}, E)$ obtained from empirical relationships$^{11,12}$ which give a maximum error margin of 6% relatively to measurements.

Including the above mechanisms and taking also into consideration $(f,\gamma)$ and $(n,\gamma)$ reactions with neutrons from all of the energy groups, Eq. (6) becomes:

$$w(\vec{r}, E)dE = dE \frac{\mu_{ab}(E)}{\mu_a(E)} (1 - e^{-\mu_a(E)\gamma})$$

$$B_{s}(\mu_a(E)|\vec{r}, E)B_{c}(\mu(E)|\vec{r} - \vec{r}_0|, E)E \frac{e^{-\mu(E)|\vec{r} - \vec{r}_0|}}{4\pi |\vec{r} - \vec{r}_0|^2} \int_{n} A_n(\vec{r}_0, E)\Phi_n(\vec{r}_0)$$

Finally, by integrating Eq. (7) over the whole photon energy spectrum and over the volume $V_c$ of the active part of the reactor core, the rate of the total gamma energy deposited per unit volume of the sample is obtained:

$$W = \int \int_{V_c} W(\vec{r}, E)dE d\vec{r}_0$$

III. APPROXIMATIONS AND DATA USED

Fission of only U-235 has been considered. If more fissile nuclides are contributing (e.g. Pu-239 and Pu-241 included at appreciable quantities in high burn-up fuel elements), then the $A_n(\vec{r}_0, E)$ quantity in Eq. (5) must be replaced by $A_n(\vec{r}_0, E) + \sum_i X_{n,i}(E)\Sigma_{f,n,i}(\vec{r}_0)$ where ‘$i’ stands for each fissile
nuclide. For the gamma spectra $X_n(E)$ and $Y_{j,n}(E)$, compiled experimental data have been utilized. For $X_n(E)$, exponential fits are used\textsuperscript{16,17}. For $Y_{j,n}(E)$, the discrete values of the PGAA-IAEA database\textsuperscript{18} have been included. It should be noted that only the gamma rays produced from reactions with thermal neutrons (fission and capture) have been considered in the present model structure, due to lack of data of gamma rays yield from epithermal neutrons reactions.

For the solution of Eq. (8), a homogenized reactor core is assumed; thus, the macroscopic cross-sections $\Sigma_{f,n}(\vec{r}_0)$ and $\Sigma_{j,n}(\vec{r}_0)$, are respectively substituted with $\Sigma_{f,n}$ and $\Sigma_{j,n}$. The attenuation coefficient $\mu(E)$ of the monoenergetic $\gamma$-rays in the homogenized core is derived as a weighted sum, with-respect-to-density, of the individual $\mu_j(E)$ values of the core materials\textsuperscript{19}. The same approximation is used for the derivation of the core dose build-up factor $B_c(\mu(E)|\vec{r} - \vec{r}_0|, E)$ based on the values $B_j(\mu(E)|\vec{r} - \vec{r}_0|, E)$ tabulated\textsuperscript{11} for each core material "j". Energy integration (8) is performed using the trapezoidal method. A 21-Point, 5\textsuperscript{th}-degree of accuracy formula for triple integrals\textsuperscript{20} is used for the volume integration.

IV. APPLICATION TO GRR-1

GRR-1 is a pool type, light water moderated and cooled reactor, using beryllium reflectors and fueled by MTR-type fuel elements. The reactor is normally operating at 5MW power. The active core dimensions in x, y (horizontal) and z (vertical) directions are 45.66cm, 47.74cm and 62.55cm respectively. There are five control blade locations in the core where shim/safety rods are placed. The horizontal configuration of the conceptual core that is used in this work is shown in Figure 1 using x (letters) and y (numbers) coordinates. The grid position D4 hosts a control fuel assembly without absorbing blade. This position is used as a flux trap, for material irradiation in the highest possible flux.

The three-dimensional group-averaged neutron flux in the GRR-1 core, $\Phi_n(\vec{r}_0)$, was calculated using the neutronics code system XSDRNPMP and CITATION-LDI2 for five neutron energy groups, the thermal threshold being at 0.53158 eV. The macroscopic cross sections of U-235 fission and
(n,γ) reactions in the core materials, Σ_{fn} and Σ_{jn} respectively, were determined with XSDRNPM assuming a homogenized core. The above neutronic calculations were performed using the NDF5-238 group library. For the thermal neutron flux required for the gamma heating computation, a calculational domain containing the reactor core shown in Figure 1, surrounded by 20cm of pool water in all six sides, was considered. For the Σ_{fn} and Σ_{jn} determination, a homogenized core was considered including only the nuclides existing in the active core, i.e., fuel, control rods, structural materials (essentially aluminum) and core water (moderator and coolant). Very good agreement was found between computed and measured thermal neutron flux (Figure 2).

Temperature measurements T(z) of a Fe cylindrical sample with 5cm height and 7mm diameter were carried out for several sample positions z along the D4 channel. The temperatures presented in Figure 2 are the equilibrium temperatures, reached when heat deposition rate equals heat losses.

The gamma heat W(z), deposited per unit volume of the above mentioned Fe cylindrical sample, placed at different depths z along channel D4, was calculated from Eq. (8) considering the γ-rays production in the active part of the reactor core. The results are also shown in Figure 2. It can be seen that the distribution of W(z) follows, to a good approximation, the distribution of the calculated (and measured) thermal neutron flux Φ(z), i.e. that the ratio W(z)/Φ(z) is, to a good approximation, independent of z. Also it can be seen that the measured temperature T(z) follows the distribution Φ(z), i.e. the ratio T(z)/Φ(z) is, to a good approximation, independent of z, which is in accordance with the finding that W(z)/Φ(z) is independent of z.

V. CONCLUSIONS

A three-dimensional numerical code of gamma heating using a point kernel parameterization was developed. The code was applied to a core configuration of the Greek Research Reactor (GRR-1). The distribution of the computed gamma heating power density follows, as expected, the distribution of the thermal neutron flux, which is in support of the code validity. Comparisons of the computed distribution of gamma heating power deposited in a Fe sample with measurements of the thermal neutron flux and the Fe sample temperatures in GRR1, show that the GHRRC code has a good
qualitative behavior. For the quantitative evaluation of the developed code, in-core gamma heating measurements will be necessary.

REFERENCES

Figure 1 Horizontal cross section of the GRR-1 Core. The notation is: F for standard fuel assemblies, CR for control fuel assemblies with control rods inserted, W for water and Be for beryllium reflectors. D4 is the irradiation channel.
Figure 2: Computed gamma heating power densities of the Fe sample (◇) and measured temperatures of the Fe sample (○), along D4 channel of GRR1. Computed (□) and measured thermal neutron flux (▲) are also depicted.