PERGAMON

Annals of Nuclear Energy 28 (2001) 1773–1782

annals of
NUCLEAR ENERGY

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Calculation of the moderator temperature coefficient of reactivity for water moderated reactors

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Received 20 November 2000; accepted 30 December 2000

Abstract

An American nuclear standard has been recently issued, providing guidance and qualitative recommendations on how the moderator temperature coefficient (MTC) of reactivity should be determined through reactor calculations in water moderated (and cooled) reactor cores (ANS, 1997. Calculation and measurement of the moderator temperature coefficient of reactivity for water moderated power reactors. American Nuclear Society, American National Standard ANSI/ANS-19.11-1997). The present work provides quantitative information on areas of concern and effects addressed in this standard, namely, the effect of the core reflector region and the effect of the magnitude of the moderator temperature change. The calculations consist of a sequence of static criticality analyses, and are performed with a purposely developed two-dimensional reactor model based on two neutron energy groups. The numerical results indicate that the thickness of the reflector has a measurable effect on the accuracy of the MTC value only for a small core, introducing uncertainties of the order of 10%. Instead, the effect of temperature change is found to be negligible within the recommended range of 3 to 5°C. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

The moderator temperature coefficient (MTC) of reactivity in water moderated reactors is an important operational parameter connected with safety considerations. The MTC is defined as the change of reactivity per degree change of the core-

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PII: S0306-4549(01)00016-0

averaged moderator temperature. As a rule, a reactor core is designed such that the MTC has a negative value. This ensures that negative reactivity feedback will be provided in the event of power excursion. However, the value of MTC should not be too negative because there exist certain off-normal sequences, in particular some cool-down accidents in PWRs, which are aggravated by a large negative MTC. Therefore, in power PWRs, limits are established on how negative MTC may become during the fuel cycle and surveillance tests are performed during the fuel cycle to determine if the MTC value complies with the specified limits. Boron dilution is generally used as a standard experimental method in such type of measurements. Noise analysis has been also contemplated as a desirable alternative method. Several investigations have been performed to assess this possibility including both experimental works (e.g. Laggiard and Runkel, 1997; Oguma et al., 1995; Herr and Thomas, 1991) and theoretical developments (Antonopoulos-Domis and Housiadas, 1999; Housiadas and Antonopoulos-Domis, 1999).

The design value of the MTC is determined with the help of reactor calculations. Besides design purposes, MTC calculations are also required in several other circumstances, for instance, to compare with measurements, or to provide input for transient calculations. The calculation of the MTC usually consists of performing a sequence of static criticality calculations for different moderator temperatures T_{M}, and then determining the MTC value from a table of k_{eff} as a function of T_{M}. In such procedure, there are several details involved in the accurate determination of the MTC value. Recently, an appropriate nuclear standard has been issued, providing guidance and qualitative recommendations on how these calculations should be performed (ANS, 1997). The objective of the present work is to provide quantitative information on areas of concern and effects addressed in the above standard. More specifically, two particular aspects are considered, namely, the effect of the core reflector region and the effect of the magnitude of the moderator temperature change.

The calculations are performed with a purposely developed reactor model. The model is based on a two-dimensional core description and accommodates two neutron energy groups. The effect of moderator temperature on the cross-sections is accommodated by developing simple algebraic expressions, rather than performing complicated group constant calculations. As it will become apparent, despite its simplicity, the overall approach is well suited for bringing out with little computational effort the influence of various calculation assumptions on the determination of the MTC value.

The paper is organized in the following way. Section 2 describes the theoretical and numerical aspects of the work. Section 3 presents the results of the numerical benchmarks and, finally, the conclusions are summarized in Section 4.

2. Modelling

The value of MTC, or a_{c}, is determined from its definition, by simply dividing the change in reactivity $\delta\rho$ due to a change δT_{M} in moderator temperature T_{M}

$$a_c = \frac{\delta \rho}{\delta T_M}. \tag{1}$$

The reactivity change $\delta\rho$ is determined as

$$\delta \rho = \frac{k_2 - k_1}{k_2 k_1} \tag{2}$$

where k_1 and k_2 are respectively the effective multiplication factors for reactor states 1 (moderator temperature T_M) and 2 (moderator temperature $T_M + \delta T_M$). The multiplication factors are determined with the help of static criticality calculations, performed with a two-dimensional, two-energy group model. Obviously, such calculations require as input the values of the macroscopic cross-sections for reactor states 1 and 2. Below, a simple model is presented for the effect of moderator temperature on the macroscopic cross-sections.

In cores moderated and cooled by water, the dominant effect of moderator temperature on reactivity is due, by far, to the effect of temperature on moderator density. The overall effect is determined by the interplay of competing trends. An increase in water temperature leads to a decrease in density, which in turn leads to a loss of moderation, thereby introducing a negative reactivity. At the same time, the reactivity tends to become more positive since a decrease in density will cause a reduction in the absorption. The latter trend is particularly favoured when appreciable chemical shim is used (e.g. use of soluble boron in PWR cores), since then, a decrease in density is translated directly to a proportional decrease in poison concentration, and hence in absorption. In a two group model, the affected cross-sections will be the moderation cross-section Σ_{12} and the absorption cross-section of the thermal group Σ_a. The absorption cross-section of the fast group Σ_{a1} can be considered as remaining unchanged because fast neutrons are mainly absorbed by virtue of capture resonances in the fuel material, rather than the moderator.

The effect on cross-section Σ_{a2} can be determined in a quiet straightforward manner because a change $\delta\rho_M$ in moderator density ρ_M induces a directly proportional change in absorption (ANS, 1997), therefore

$$\delta \Sigma_{a2} \sim \delta \rho_M \tag{3}$$

where $\delta\Sigma_{a2}$ is the change in cross-section Σ_{a2}. On account of the definition of the fluid (moderator) expansion coefficient β_M

$$\beta_M \triangleq \frac{1}{u_M} \frac{\mathrm{d}u_M}{\mathrm{d}T_M} \tag{4}$$

where u_M is specific volume, $u_M = 1/\rho_M$, one can readily show that (3) leads to

$$\delta \Sigma_{a2} = -\beta_M \Sigma_{a2} \delta T_M \tag{5}$$

which is the relationship sought, giving the effect of temperature on the absorption cross-section of the thermal group.

The effect on moderation cross-section Σ_{12} arises from the induced effect on resonance absorption (Duderstadt and Hamilton, 1976). The resonance escape probability p can be written as

$$p = \exp\left(-\frac{\rho_F}{\xi\rho_M\sigma}I\right) \tag{6}$$

where ρ_F is the fuel density, ξ the mean lethargy gain per collision, σ the microscopic scattering cross-section, and I the resonance integral. By assuming all parameters in (6) to be temperature independent except moderator density, and following differentiation with respect to T_M, one obtains

$$\frac{dp}{dT_M} = p\ln(1/p)\frac{1}{\rho_M}\frac{d\rho_M}{dT_M}. \tag{7}$$

In view of definition (4), the above expression reads

$$\frac{dp}{dT_M} = -\beta_M p\ln(1/p). \tag{8}$$

On the other hand, the well-known expression for the resonance escape probability

$$p = \frac{\Sigma_{12}}{\Sigma_{12} + \Sigma_{a1}} \tag{9}$$

implies that

$$\delta\Sigma_{12} = \frac{\Sigma_{12}}{p(1-p)}\delta p \tag{10}$$

which considering (8), gives the required relationship connecting the variations between moderator temperature and cross-section of moderation

$$\delta\Sigma_{12} = -\beta_M\frac{\ln(1/p)}{1-p}\Sigma_{12}\delta T_M. \tag{11}$$

Obviously, Eqs. (5) and (11) enable the effect of moderator temperature on the macroscopic cross-sections to be numerically expressed in a simple and efficient way, without the need of undertaking involved group constant calculations.

The reactor model that was developed in the present work is two-dimensional and consists of a homogenous multiplying core, surrounded by a non-multiplying reflector. The effective multiplication factor is determined with the help of steady-state critically calculations using the two-energy group diffusion equations

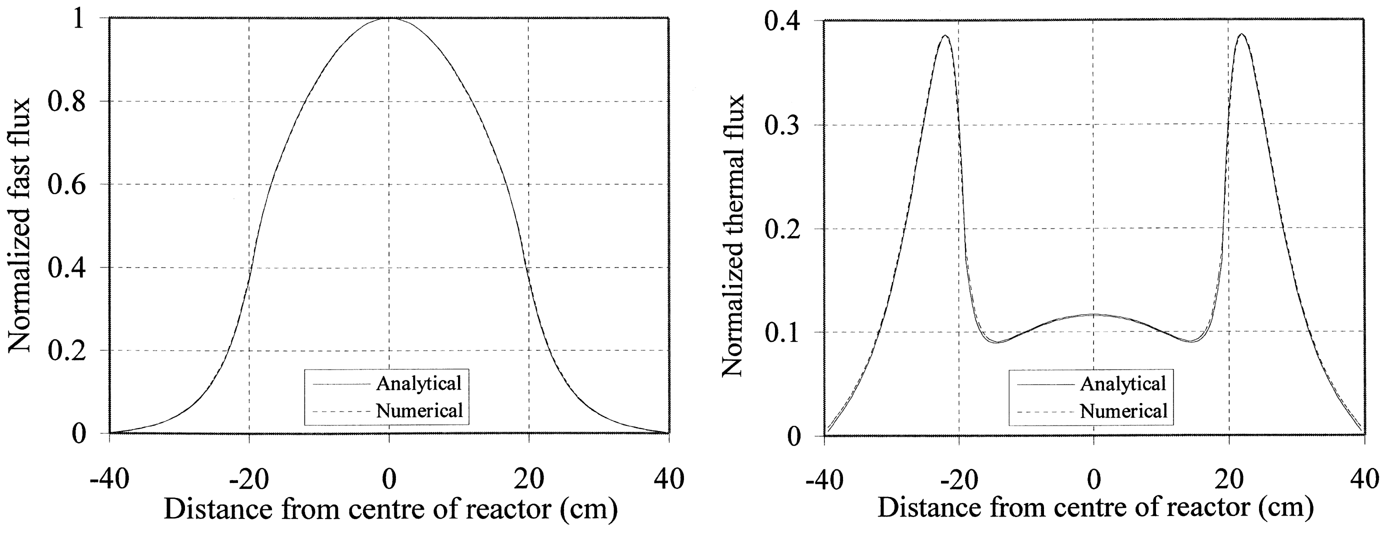
$$
\begin{aligned}
&-\nabla D_1(x,y)\nabla\Phi_1(x,y) + (\Sigma_{a1} + \Sigma_{12})\Phi_1(x,y) = \frac{1}{k}[\nu_1\Sigma_{f1}\Phi_1(x,y) + \nu_2\Sigma_{f2}\Phi_2(x,y)] \\
&-\nabla D_2(x,y)\nabla\Phi_2(x,y) + \Sigma_{a2}\Phi_2(x,y) = \Sigma_{12}\Phi_1(x,y) \qquad (12)
\end{aligned}
$$

where the symbols have their usual meaning. The above differential equations are solved over a rectangular uniform two-dimensional mesh using finite differences. The resulting algebraic equations constitute an eigenvalue problem for k. The power method was used, which is a well-known iterative method for the determination of the fundamental eigenfunctions (i.e. fluxes Φ_1 and Φ_2), corresponding to the largest eigenvalue, which is k_{eff}. The required inner iterations are performed using the successive overrelaxation (SOR) method. The convergence criterion for k_{eff} was set equal to 1×10^{-8}, which is far more stringent in comparison to the value of 5×10^{-6} recommended by the nuclear standard (ANS, 1997). Further details on the employed procedure can be found in standard texts (e.g. Duderstadt and Hamilton, 1976).

The numerical code was validated by comparing with known analytical solutions, as available for the case of a one-dimensional core with reflector on both sides (Lamarsh, 1966). A two-dimensional reactor code may generate one-dimensional predictions when $H/W \gg 1$ where H is reactor height and W reactor width. Accordingly, the assessment exercise was performed by considering a reactor with $H = 800$ cm and $W = 80$ cm. Fig. 1 compares the computed fluxes to the analytical solution. As can be seen the agreement is excellent, providing thus the required evidence for proper operation of the code.

3. Results

The calculations have been performed using as input data the following typical PWR core data (Duderstadt and Hamilton, 1976): diffusion coefficient of fast neutrons $D_1 = 1.2627$ cm, diffusion coefficient of thermal neutrons $D_2 = 0.3543$ cm, cross-section of absorption in the fast group $\Sigma_{a1} = 0.1207$ cm^{-1}, cross-section of absorption in the thermal group $\Sigma_{a2} = 0.1210$ cm^{-1}, cross-section of moderation $\Sigma_{12} = 0.01412$ cm^{-1}, fission source in the fast group $\nu_1\Sigma_{f1} = 0.008476$ cm^{-1}, fission source in the thermal group $\nu_2\Sigma_{f2} = 0.18514$ cm^{-1}. The above data are used to determine the state 1 effective multiplication factor k_1. Then, the state 2 effective multiplication factor k_2 is evaluated by keeping all parameters the same besides cross-sections Σ_{a2} and Σ_{12}, which are updated to $\Sigma_{a2} + \delta\Sigma_{a2}$ and $\Sigma_{12} + \delta\Sigma_{12}$, respectively, with the help of expressions (5) and (11). A moderator temperature change of $\delta T_M = 4°$C is considered to be the reference increment value. Also, two



(a) (b)

Fig. 1. Comparison between code generated and analytically evaluated fast (a) and thermal (b) neutron fluxes for a one-dimensional core reflected on both sides (core width 40 cm, reflector thickness 20 cm).;; >

core sizes have been considered. A large core having a height of $H = 370$ cm and a width of $W = 340$ cm, and a small core with $H = W = 100$ cm.

The effect of the reflector is shown in Figs. 2 and 3 for the cases of the large and small core, respectively. The calculations have been performed using $\delta T_{\mathrm{M}} = 4°\mathrm{C}$. The results are shown in terms of the %-variation in the calculated MTC, as function of the thickness of the reflector. The MTC value corresponding to a bare core (zero thickness of reflector) is used as the reference value. The reference values correspond to an MTC of $-43.4\times10^{-5}\ \mathrm{K}^{-1}$ for the large core and $-48\times10^{-5}\ \mathrm{K}^{-1}$ for the small core. Note that the calculated values are consistent with the MTC values typically encountered in PWR reactor cores. As can be seen in Fig. 2, the effect for the large core, although manifest, is practically negligible ($<1\%$). Instead, the effect is significantly magnified in the case of small core, becoming of about 7%. One may remark that the effect is enhanced at small reflector thickness, while at larger reflector sizes a plateau is reached. This trend is quite expected, considering that passed a reflector thickness of some diffusion lengths, the reflector behaves practically as of infinite size.

The effect of the moderator temperature change δT_{M} is shown in Fig. 4. The investigated range is selected to include the range of $3 \leqslant \delta T_{\mathrm{M}} \leqslant 5°\mathrm{C}$, as recommended in the nuclear standard (ANS, 1997). This recommendation is based on a compromise between two competing considerations. A δT_{M} that is too small will produce a reactivity change that is determined more by interpolation accuracy and round-off, whereas, a δT_{M} that is too large will produce an MTC that would not reflect the value of the moderator temperature of interest. In Fig. 4 only the large core results are shown. The results obtained with the small core are practically the same, and

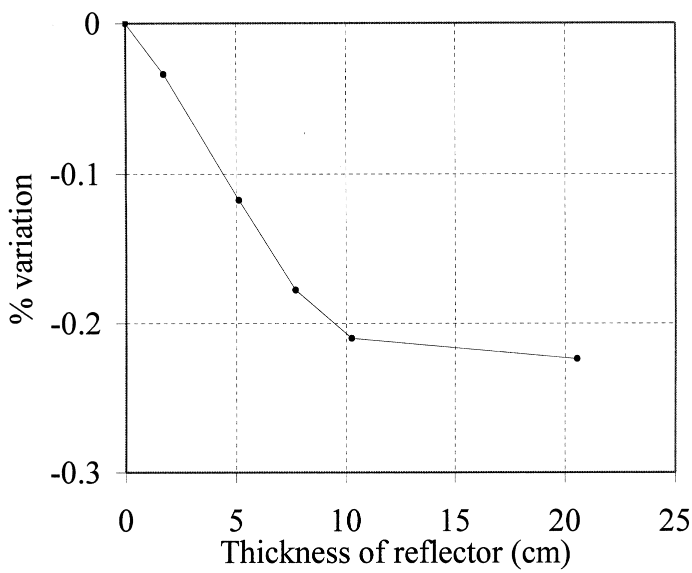


Fig. 2. Percentage variation of MTC as a function of thickness of reflector for a large core ($H = 370$ cm, $W = 340$ cm).;; >

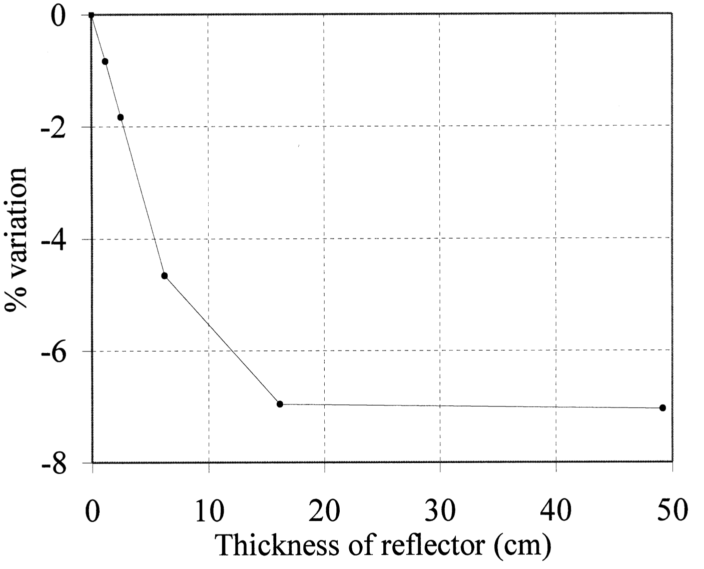


Fig. 3. Percentage variation of MTC as a function of thickness of reflector for a small core ($H = 100$ cm, $W = 100$ cm).;; >

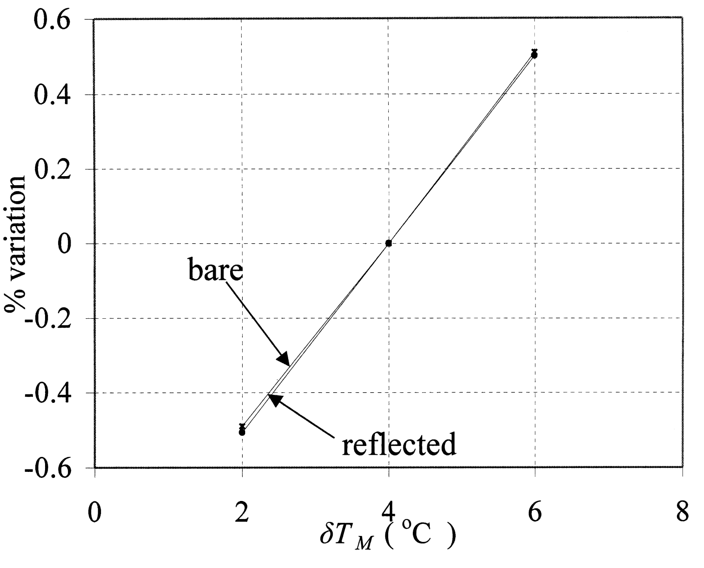


Fig. 4. Percentage variation of MTC as a function of moderator temperature change δT_M.&#&#2;; >

therefore are not reproduced here. As before, the effect is given in terms of %-variation in the calculated MTC, as function of the temperature change δT_M. Here again, the reference MTC value is considered that obtained with $\delta T_M = 4$°C. As the results indicate, the effect of δT_M induces uncertainties $\leqslant 0.5$%with respect to the reference MTC value. Therefore, the effect can be considered as negligible. This result is not affected by the presence of a reflector. Indeed, as shown in Fig. 4, the same trends are obtained, regardless of considering a bare or a reflected reactor core.

4. Conclusions

The present work investigated the influence of calculation/modelling assumptions on the accuracy of the determination of the moderator temperature coefficient (MTC) of reactivity in water moderated reactors. The emphasis was placed on two particular concerns addressed in a recent nuclear standard (ANS, 1997) on the calculation of the MTC. More specifically, the effect of the reflector and the selection of a moderator temperature change are considered. The major findings are summarized below.

1. The thickness of the reflector has a measurable effect on the accuracy of the MTC value only for a small core, introducing uncertainties of the order of 7%.
2. For a large core, the MTC calculation remains practically unaffected by the presence (or not) of a reflector.
3. The selection of a temperature change has no effect on the calculated MTC value within the recommended range of 3 to 5°C.

To perform the necessary criticality calculations a two-dimensional numerical reactor model (static) has been developed and validated, based on two-group diffusion theory. An analysis has been also made to determine the effect of moderator temperature on the macroscopic cross-section data. The analysis permitted to develop simple working expressions [Eqs. (5) and (11)], thereby circumventing the need to perform complicated group constant calculations.

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